LESTER C. NOECKER SCHOOL
ROSELAND SCHOOL DISTRICT

Math Curriculum

Kindergarten

Updated and aligned NJ SLS

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Giuseppe Leone, Superintendent
With special appreciation to Michele Smith & the Math Curriculum Writing Staff

Roseland Mathematics

Grade Level: Kindergarten

Contents

Instructional Materials

Everyday Mathematics 4th Edition
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Supplemental Resources

- Connected Ed https://connected.mcgraw-hill.com/connected/login.do
- Illustrative Mathematics https://www.illustrativemathematics.org/
- Khan Academy https://www.khanacademy.org/
- Math for Elementary School Teachers http://www.mathforelementaryteachers.org/ video clips that contain explanations of arithmetic topics including: Place Value/Arithmetic Models/Arithmetic Algorithms, Mental Math, Primes/Divisibility, Fraction Arithmetic, and Word Problems/Model Drawing.
- National Council of Teachers of Mathematics http://www.nctm.org/
- National Library of Virtual Manipulatives http://nlvm.usu.edu/

- NCTM Illuminations Resources for Teaching Math http://illuminations.nctm.org/

Interdisciplinary Connections

Mathematics is a unified body of knowledge whose concepts build upon each other. Connecting mathematical concepts includes linking ideas to related ideas learned previously.

Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas. Students need to connect their mathematical learning to appropriate real-world contexts. They need to create interest and maintain the interest after the novelty of the work has worn off.

Mathematics is the language of science and is greatly utilized in industry and business. It gives us the power to solve difficult real-world problems, but also helps us to understand how the universe operates.

Every mathematics teacher needs to make students unafraid of the subject by convincing the students of the usefulness of learning mathematics in their daily lives and for higher studies. The world today, which leans more and more heavily on Science and Technology, demands more from mathematics. Tomorrow's world will, no doubt, make still greater demands from mathematics.

Interdisciplinary Connections for Kindergarten

Literature:
~ *The Greedy Triangle*, By Meghan Everette
- Introduce the lesson by reading *The Greedy Triangle*
- Follow the lesson in the Interdisciplinary Supplemental Section.

~ *Ten Little Monkeys: Jumping on the Bed* by Annie Kubler
- Introduce the lesson by reading *Ten Little Monkeys: Jumping on the Bed.*
- Follow the lesson in the Interdisciplinary Supplemental Section.

~Dr. Seuss Math
- Follow the worksheets in the Interdisciplinary Supplemental Section

New Jersey Student Learning Standards (NJSLS)

In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Counting and Cardinality

Know number names and the count sequence.

K.CC.A.1 Count to 100 by ones and by tens.

K.CC.A.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Understandings	Essential Questions
Students will understand that… ● counting involves one-to-one correspondence. ● one can count by different amounts (ones, tens, etc.).	● Why do we need to count? ● How do we count?
Knowledge	**Skills**
Students will know… ● multiples of ten. ● how to count. ● how to write the numerals 0 – 9. ● count a number of objects.	Students will be able to . . . ● count to 100 by ones. ● count to 100 by tens. ● count forward beginning from a given number within the known sequence. ● write numbers from 0-20. ● represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

RESOURCES

- **Everyday Mathematics 4 Routines:** 1, 3; **Lessons** 1-4, 1-5, 1-6, 2-4, 2-5, 2-6, 2-11, 3-1, 3-2, 3-4, 3-5, 3-6, 3-7, 3-8, 3-9, 3-10, 3-11, 3-12, 3-13, 4-1, 4-2, 4-4, 4-5, 4-6, 4-7, 4-8, 4-9, 4-11, 4-12, 4-13, 5-1, 5-2, 5-4, 5-6, 5-7, 5-8, 5-9, 5-11, 5-12, 5-13, 6-3, 6-4, 6-7, 6-11, 6-12, 6-13, 7-1, 7-2, 7-3, 7-5, 7-7, 7-8, 7-9, 7-10, 7-11, 7-12, 7-13, 8-1, 8-3, 8-4, 8-5, 8-6, 8-10, 8-12, 9-3, 9-8, 9-12, 9-13
- **Supplemental Lessons:** Binder pages 2-59, 60-61, 64-98, 99-101, 105-106

Count to tell the number of objects.

K.CC.B.4 Understand the relationship between numbers and quantities to 10; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
c. Understand that each successive number name refers to a quantity that is one larger.

K.CC.B.5 Count to answer "how many?" questions about as many as 10 things arranged in a line, a rectangular array, or a circle, or as many as 5 things in a scattered configuration; given a number from 1–10, count out that many objects.

Understandings	Essential Questions
Students will understand that: ● there is a relationship between the numbers and quantities. ● when counting, each object has one and only one number name and each number name is paired with one and only one object (one-to-one correspondence). ● when counting, the last number name said tells the number of objects counted. ● the number of objects is the same regardless of the order in which they were counted. ● each successive number name refers to a quantity that is one larger.	● How do we count?
Knowledge	**Skills**
Students will know… ● the connection between counting and cardinality. ● one-to-one correspondence.	Students will be able to . . . ● count objects while saying the number names in the standard order. ● state the total number of objects in a group. ● count as many as 20 things arranged in a line, a rectangular array, or a circle, when asked "how many…?". ● count as many as 10 things in a scattered configuration, when asked "how many…?". ● count out the correct number of objects when given a number from 1-20.

RESOURCES

- **Everyday Mathematics 4 Routines**: 1-5; **Lessons** 1-3, 1-5 to 1-13, 2-1 to 2-4, 2-6, 2-8 to 2-11, 2-13, 3-1, 3-2, 3-4 to 3-11, 3-13, 4-1 to 4-4, 4-7, 4-8, 5-1 to 5-3, 5-6 to 5-11, 5-13, 6-3 to 6-7, 6-11, 6-13, 7-2, 7-3, 7-5, 7-7 to 7-9, 7-11, 8-1, 8-4 to 8-6, 8-9, 8-10, 9-1, 9-3, 9-12, 9-13
- **Supplemental Lessons:** Binder pages 1-23, 39-59, 61-63, 99-101, 106

Counting and Cardinality

Compare numbers.

K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies (include groups with up to ten objects).

K.CC.C.7 Compare two numbers between 1 and 10 presented as written numerals.

Understandings	Essential Questions
Students will understand that… ● "greater than" means the amount is more; "less than" means the amount is less. ● a numeral stands for number of concrete objects.	● How can we compare two numbers?
Knowledge	**Skills**
Students will know… ● matching strategies to identify the number of objects in a group of up to 10 objects. ● counting strategies to identify the number of objects in a group of up to 10 objects.	Students will be able to . . . ● identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. ● compare two numbers between 1 and 10 presented as written numerals.

RESOURCES

- **Everyday Mathematics 4 Routines**: 2, 4, 5; **Lessons** 1-7, 1-8, 2-1, 2-2, 2-6, 2-10, 3-1, 3-7, 3-11, 3-12, 4-1, 4-3, 4-6, 4-8, 4-12, 5-3, 5-7, 5-8, 5-9, 5-12, 6-3, 6-5, 6-6, 6-9, 6-12, 6-13, 7-2, 7-3, 7-7, 7-8, 7-9, 7-10, 7-12, 8-3, 8-5, 8-6, 8-10, 8-11, 8-13, 9-1, 9-2, 9-3, 9-4, 9-5, 9-8, 9-9, 9-12
- **Supplemental Lessons:** Binder pages 1, 61, 102-104, 106

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.A.1 Represent addition and subtraction up to 10 with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

K.OA.A.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

K.OA.A.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

K.OA.A.5 Demonstrate fluency for addition and subtraction within 5.

Understandings	Essential Questions
Students will understand that… ● numbers can be decomposed. ● making a sum of 10 will be important to make work easier. ● objects, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations can help one understand problems and find solutions.	● Why do we need to add and subtract? ● What happens when we put groups together or add to a group? ● What happens when we take apart groups or take away from a group?
Knowledge	**Skills**
Students will know… ● addition and subtraction can be represented in multiple ways. ● numbers can be decomposed.	Students will be able to . . . ● represent addition and subtraction with objects, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. ● solve addition and subtraction word problems. ● add and subtract within 10. ● decompose numbers less than or equal to 10 into pairs in more than one way by using objects or drawings. ● record decompositions of numbers by a drawing or equation (5 = 4 + 1). ● find the number that makes 10 when added to a given number, for any number 1-9, by using objects or drawings and record the answer with a drawing or an equation. ● fluently add and subtract within 5.
RESOURCES	

- **Everyday Mathematics 4 Routines**: 2, 3, 5; **Lessons** 1-9, 1-10, 1-11, 2-4, 2-5, 2-8, 2-9, 2-12, 2-13, 3-2, 3-3, 3-9, 3-12, 4-5, 4-8, 4-9, 5-2, 5-3, 5-5, 5-6, 5-7, 5-9, 5-10, 5-11, 5-13, 6-4, 6-7, 6-8, 6-9, 6-10, 6-11, 6-12, 6-13, 7-1, 7-2, 7-4, 7-5, 7-6, 7-7, 7-9, 7-10, 7-12, 7-13, 8-2, 8-4, 8-5, 8-7, 8-8, 8-9, 8-11, 8-12, 8-13, 9-2, 9-3, 9-5, 9-6, 9-7, 9-9, 9-10, 9-11, 9-12, 9-13
- **Supplemental Lessons:** Binder pages 61-63, 84-98, 99-101, 105-108

Numbers and Operation in Base 10

Work with numbers 11-19 to gain foundations for place value.

K.NBT.A.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Understandings	Essential Questions
Students will understand that... ● teen numbers (11-19) are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	● Why do we compose and decompose numbers?
Knowledge	**Skills**
Students will know... ● composing and decomposing numbers into tens and ones will help solve problems	Students will be able to . . . ● compose and decompose numbers from 11-19 into ten ones and some further ones, by using objects or drawings. ● record compositions and decompositions with drawings or equations.

RESOURCES

- **Everyday Mathematics 4 Routines**: 1, 3; **Lessons** 5-6, 5-8, 6-3, 6-12, 6-13, 7-3, 8-6, 8-13

Measurement

Describe and compare measurable attributes.

K.M.A.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

K.M.A.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Understandings	Essential Questions
Students will understand that… ● measurable attributes are a way to compare objects. ● an object may have multiple measurable attributes. ● multiple objects may have the same measurable attribute.	● Why do we need to measure objects? ● What attributes are measurable? ● How do we compare objects?
Knowledge	**Skills**
Students will know… ● the potential attributes are measurable. ● words that compare attributes.	Students will be able to . . . ● describe measurable attributes of objects, such as length or weight. ● describe several measurable attributes of a single object. ● directly compare two objects with a measurable attribute in common to determine which has "more of"/"less of" the attribute, e.g. heights of two children. ● describe the difference between two objects with the common attribute that was compared, one child is taller/shorter than the other child.

RESOURCES

- **Everyday Mathematics 4 Lessons:** 1-1, 1-4, 2-7, 3-5, 3-10, 4-1, 4-9, 4-10, 4-13, 5-1, 6-1, 6-2, 6-6, 6-7, 6-10, 7-1, 7-6, 7-8, 7-13, 8-3, 9-4, 9-5, 9-8, 9-9, 9-12, 9-13
- **Supplemental Lessons:** Binder page 105

Measurement

Work with money.

K.M.B.3 Understand that certain objects are coins and dollar bills, and that coins and dollar bills represent money. Identify the values of all U.S. coins and the one-dollar bill.

Understandings	Essential Questions
Students will understand that… ● different coins have unique values ● the relative value of the coins are not related to the relative values of the coins: i.e. a penny is larger than a dime but it is not worth more than a dime ● coins can be identified by their color, size and edge ● coins have 2 sides: i.e. a front and back, heads and tails ● money can be counted and compared ● some coins can be exchanged for other coins: e.g. 5 pennies can be exchanged for 1 nickel ● the value of some coins and bills can be represented by a combination of other coins	● Why do we need money? ● How do we count money?
Knowledge	**Skills**
Students will know… ● pennies are copper and nickels, dimes and quarters are silver. ● the size order of coins from small to large: dime, penny, nickel, quarter ● pennies and nickels have a smooth edge while dimes and quarters have an edge with ridges.	Students will be able to . . . ● identify a penny, nickel, dime and quarter ● sort coins ● identify the value of a penny, nickel, dime, and quarter ● skip count to count one type of coin: 1s, 5s, 10s, i.e. 1, 2, 3, 4… for pennies i.e. 5, 10, 15… for nickels i.e. 10, 20, 30… for dimes i.e. 25, 50, 75, 100 for quarters

RESOURCES

- **Daily Mathematics Routines:** calendar, counting, skip counting
- **Everyday Mathematics Lessons:** 4.11 Counting by 10s (dimes)
- **Supplemental Lessons:** Binder pages 120- 135

Data Literacy

Classify objects and count the number of objects in each category.

K.DL.A.1 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. *Limit category counts to be less than or equal to 10.*

Understandings	Essential Questions
Students will understand that… ● classifying objects helps to count total numbers. ● objects can be described by their attributes. ● objects can be sorted by their attributes.	● Why do we need to classify objects? ● How does sorting help us to count?
Knowledge	**Skills**
Students will know… ● attributes that can be used to sort or classify objects.	Students will be able to . . . ● classify objects into given categories. ● count the number of objects in a category (counts less than or equal to 10). ● sort the categories by count.

RESOURCES

- **Everyday Mathematics 4 Routines**: 2, 4, 5; **Lessons** 1-7, 1-8, 2-7, 2-10, 3-1, 4-1, 4-3, 5-3, 6-3, 6-5, 6-6, 7-2, 7-7, 7-9, 7-13, 8-6, 9-1, 9-12
- **Supplemental Lessons:** Binder pages 1, 102-104, 106

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cylinders, and spheres).

K.G.A.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.

K.G.A.2 Correctly name shapes regardless of their orientations or overall size.

K.G.A.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Understandings	Essential Questions
Students will understand that… ● shapes have positions in the world relative to other things.	

● characteristics of shapes give it a name.	● What characteristics of a shape help us to name it? ● How does knowing the name of shapes help us? ● Why do we need to know positions of shapes?
Knowledge	**Skills**
Students will know… ● the characteristics of a square, circle, triangle, rectangle, hexagon, cube, cylinder, and sphere. ● the meaning of the words above, below, beside, in front of, behind, and next to.	Students will be able to . . . ● Describe objects in the environment using names of shapes. ● Describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to*.
RESOURCES	
● **Everyday Mathematics 4 Lessons:** 1-2, 1-5, 1-13, 2-3, 2-7, 2-8, 2-11, 2-12, 3-3, 3-6, 3-11, 3-13, 4-1, 4-2, 4-7, 4-8, 4-10, 4-11, 5-4, 5-5, 5-11, 5-13, 6-1, 6-4, 6-5, 6-8, 6-9, 6-10, 7-1, 7-4, 7-8, 7-11, 7-13, 8-1, 8-2, 8-3, 8-7, 8-8, 8-11, 9-1, 9-4, 9-6, 9-7, 9-10, 9-12, 9-13 ● **Supplemental Lessons:** Binder pages 60-61, 64-83, 99-101, 102-105, 107-108	

Geometry	
Analyze, compare, create, and compose shapes. **K.G.B.4** Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). **K.G.B.5** Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. **K.G.B.6** Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	
Understandings	**Essential Questions**
Students will understand that… ● shapes in the world can be built with components such as sticks and clay balls. ● shapes in the world can be drawn. ● shapes can be formed by composing other shapes.	● Why do we need to identify shapes? ● Why would we compose shapes?
Knowledge	**Skills**
Students will know…	Students will be able to . . .

• the characteristics of a square, circle, triangle, rectangle, hexagon, cube, cylinder, and sphere. • components/representations that can be used to model shapes in the world.	• analyze two- and three-dimensional shapes, using informal language. • compare two- and three-dimensional shapes, using informal language. • model shapes in the world by building shapes from components and drawing shapes.

RESOURCES

- **Everyday Mathematics 4 Lessons:** 1-2, 1-5, 1-12, 2-2, 2-3, 2-7, 2-8, 2-11, 2-12, 3-3, 3-11, 4-2, 4-7, 4-8, 4-10, 4-11, 5-4, 5-11, 5-13, 6-4, 6-5, 6-8, 6-9, 6-10, 7-1, 7-4, 7-11, 7-13, 8-1, 8-2, 8-3, 8-7, 8-8, 9-1, 9-4, 9-6, 9-7, 9-10, 9-12, 9-13
- **Supplemental Lessons:** Binder pages 61, 64-83, 99-105, 107-108

Additional Lessons for Kindergarten

Measurement

Although not required in the standards, students need to be exposed to additional topics in order to prepare for what is required in future grades. These topics in Kindergarten include **MONEY**, time, and patterns.

Understandings	Essential Questions
Students will understand that… • different coins have unique values. • the relative sizes of the coins are not related to the relative values of the coins (i.e., a penny is larger than a dime but it is not worth more than a dime.) • some coins can be exchanged for other coins, e.g., 5 pennies can be exchanged for 1 nickel. • the value of some coins and bills can be represented by a combination of other coins. • money amounts can be counted and compared. • coins can be identified by their color, size, and edge.	• Why do we need money? • How do we count money?
Knowledge	**Skills**
Students will know… • pennies are copper and nickels, dimes, and quarters are silver.	Students will be able to . . . • identify a penny, nickel, dime, and quarter. • sort coins. • identify the value of a penny, nickel, dime, and quarter.

● a nickel is bigger than a dime but smaller than a quarter. ● pennies and nickels have a smooth edge while dimes and quarters have an edge with ridges.	● skip count to count one type of coin, e.g., 10, 20, 30 for dimes. ●
RESOURCES	
● Done in morning routines. ● **Supplemental Lessons:** Binder pages 109-123	

Additional Lessons for Kindergarten

Measurement

Although not required in the standards, students need to be exposed to additional topics in order to prepare for what is required in future grades. These topics in Kindergarten include money, **TIME**, and patterns.

Understandings	Essential Questions
Students will understand that… ● some activities take more time than others to complete. ● a day has three parts that we discuss: morning, afternoon, and evening. ● when time passes, the hour hand and the minute hand move at different rates. ● the hour hand represents the approximate time of the day, the minute hand gives a more exact time. ● events happen in order- we use terms such as first, next, and last.	● Why do we need clocks? ● What are the different types of clocks? ● How do we tell time?
Knowledge	**Skills**
Students will know… ● there are two cycles to the passage of time, 12:00 through 11:59, during the 24 hours of a day. ● the hour hand must be pointing at the number exactly for it to be “o’clock.”	Students will be able to . . . ● identify the part of day, morning, afternoon, evening. ● recognize the numbers 1-12 on the face of a clock. ● tell time to the hour WITH THE HOUR HAND ONLY.

RESOURCES

Done in morning routines.

Additional Lessons for Kindergarten

Algebraic Thinking

Although not required in the standards, students need to be exposed to additional topics in order to prepare for what is required in future grades. These topics in Kindergarten include money, time, and **PATTERNS**.

Understandings	Essential Questions
Students will understand that… ● the same set of objects can be used to create different patterns. ● some patterns are made up of units that repeat. ● some patterns can be identified by type, e.g., ABABAB. ● many things can be used to create patterns, e.g., shapes, colors, sounds, letters, and objects.	● Why do we need to identify patterns? ● How do we recognize a pattern?
Knowledge	**Skills**
Students will know… ● some common patterns types, e.g. ABABAB…; clap, clap, stomp, clap, clap, stomp….	Students will be able to . . . ● recognize patterns. ● create patterns. ● extend a given pattern.

RESOURCES

Done in morning routines.

Connecting the Standards for Mathematical Content to the Standards for Mathematical Practice

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for

an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards, which set an expectation of understanding, are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation
$(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding
$(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Standard 9
21st Century Life and Careers

In today's global economy, students need to be lifelong learners who have the knowledge and skills to adapt to an evolving workplace and world. To address these demands, Standard 9, 21st Century Life and Careers, which includes the 12 Career Ready Practices, establishes clear guidelines for what students need to know and be able to do in order to be successful in their future careers and to achieve financial independence.

Mission: *21st century life and career skills enable students to make informed decisions that prepare them to engage as active citizens in a dynamic global society and to successfully meet the challenges and opportunities of the 21st century global workplace.*

Vision: To integrate 21st Century life and career skills across the K-12 curriculum and in Career and Technical Education (CTE) programs to foster a population that:

- Continually self-reflects and seeks to improve the essential life and career practices that lead to success.
- Uses effective communication and collaboration skills and resources to interact with a global society.
- Is financially literate and financially responsible at home and in the broader community.
- Is knowledgeable about careers and can plan, execute, and alter career goals in response to changing societal and economic conditions.
- Seeks to attain skill and content mastery to achieve success in a chosen career path.

The Standards: Standard 9 is composed of the Career Ready Practices and Standard 9.1, 9.2, and 9.3 which are outlined below:

- **The 12 Career Ready Practices**
  These practices outline the skills that all individuals need to have to truly be adaptable, reflective, and proactive in life and careers. These are researched practices that are essential to career readiness.
- **9.1 Personal Financial Literacy**
  This standard outlines the important fiscal knowledge, habits, and skills that must be mastered in order for students to make informed decisions about personal finance. Financial literacy is an integral component of a student's college and career readiness, enabling students to achieve fulfilling, financially-secure, and successful careers.
- **9.2 Career Awareness, Exploration, and Preparation**
  This standard outlines the importance of being knowledgeable about one's interests and talents, and being well informed about postsecondary and career options, career planning, and career requirements.
- **9.3 Career and Technical Education**
  This standard outlines what students should know and be able to do upon completion of a CTE Program of Study.

For students to be college and career ready they must have opportunities to understand career concepts and financial literacy. This includes helping students make informed decisions about their future personal, educational, work, and financial goals. By integrating Standard 9 into instruction, New Jersey students will acquire the necessary academic and life skills to not only achieve individual success but also to contribute to the success of our society.

21st Century Themes

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

CRP1. Act as a responsible and contributing citizen and employee.

CRP2. Apply appropriate academic and technical skills.

CRP3. Attend to personal health and financial well-being.

CRP4. Communicate clearly and effectively and with reason.

CRP5. Consider the environmental, social and economic impacts of decisions.

CRP6. Demonstrate creativity and innovation.

CRP7. Employ valid and reliable research strategies.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP9. Model integrity, ethical leadership and effective management.

CRP10. Plan education and career paths aligned to personal goals.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

CRP1. **Act as a responsible and contributing citizen and employee**
Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.

CRP2. Apply appropriate academic and technical skills.
Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation

CRP3. Attend to personal health and financial well-being.
Career-ready individuals understand the relationship between personal health, workplace performance and personal well-being; they act on that understanding to regularly practice healthy diet, exercise and mental health activities. Career-ready individuals also take regular action to contribute to their personal financial wellbeing, understanding that personal financial security provides the peace of mind required to contribute more fully to their own career success.

CRP4. Communicate clearly and effectively and with reason.
Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

CRP5. Consider the environmental, social and economic impacts of decisions.
Career-ready individuals understand the interrelated nature of their actions and regularly make decisions that positively impact and/or mitigate negative impact on other people, organization, and the environment. They are aware of and utilize new technologies, understandings, procedures, materials, and regulations affecting the nature of their work as it relates to the impact on the social condition, the environment and the profitability of the organization.

CRP6. Demonstrate creativity and innovation.
Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.

CRP7. Employ valid and reliable research strategies.
Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

CRP9. Model integrity, ethical leadership and effective management.
Career-ready individuals consistently act in ways that align personal and community-held ideals and principles while employing strategies to positively influence others in the workplace. They have a clear understanding of integrity and act on this understanding in every decision. They use a variety of means to positively impact the directions and actions of a team or organization, and they apply insights into human behavior to change others' action, attitudes and/or beliefs. They recognize the near-term and long-term effects that management's actions and attitudes can have on productivity, morals and organizational culture.

CRP10. Plan education and career paths aligned to personal goals.
Career-ready individuals take personal ownership of their own education and career goals, and they regularly act on a plan to attain these goals. They understand their own career interests, preferences, goals, and requirements. They have perspective regarding the pathways available to them and the time, effort, experience and other requirements to pursue each, including a path of entrepreneurship. They recognize the value of each step in the education and experiential process, and they recognize that nearly all career paths require ongoing education and experience. They seek counselors, mentors, and other experts to assist in the planning and execution of career and personal goals.

CRP11. Use technology to enhance productivity.
Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.

CRP12. Work productively in teams while using cultural global competence.
Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

Differentiation Strategies

Students with Disabilities/ Students at Risk of School Failure

(For students with disabilities, appropriate accommodations, instructional adaptations, and/or modifications should be determined by the IEP or 504 team)

Modifications for Classroom

- Pair visual prompts with verbal presentations
- Ask students to restate information, directions, and assignments.
- Give repetition and practice exercises
- Model skills/techniques to be mastered
- Give extended time to complete class work
- Provide copy of class notes
- Determine if preferential seating would be beneficial
- Provide access to a computer
- Provide copies of textbooks for home
- Provide access to books on tape/CD/digital media, as available and appropriate
- Assign a peer helper in the class setting
- Provide oral reminders and check student work during independent work time
- Assist student with long and short term planning of assignments
- Encourage student to proofread assignments and tests
- Provide regular parent/school communication

Modifications for Homework and Assignments

- Provide extended time to complete assignments
- Break down assignments
- Provide the student with clearly stated (written) expectations and grading criteria for assignments
- Implement RAFT activities as they pertain to the types/modes of communication (role, audience, format, topic)

Modifications for Assessments

- Provide extended time on classroom tests and quizzes
- Provide alternate setting as needed
- Restate, reread, and clarify directions/questions
- Distribute study guide for classroom tests
- Establish procedures for accommodations /modifications for assessments

Differentiation Strategies

Gifted and Talented

(content, process, product and learning environment)

- Allow students to pursue independent projects based on their individual interests
- Provide enrichment activities that include more advanced material
- Allow team-teaching opportunities and collaboration
- Set individual goals
- Conduct research and provide presentation of appropriate topics
- Design surveys to generate and analyze data to be used in discussion.
- Use Higher-Level Questioning Techniques
- Provide assessments at a higher level of thinking

English Language Learners

Modifications for Classroom

- Pair visual prompts with verbal presentations
- Provide repetition and practice
- Model skills/techniques to be mastered

Modifications for Homework/Assignments

- Provide Native Language Translation (peer, online assistive technology, translation device, bilingual dictionary)
- Provide extended time for assignment completion as needed
- Highlight key vocabulary
- Use graphic organizers